**Probability:**

**Random Variables:**

Probability involves sample space (Ω) and an event (E)

Event is a subset of the sample space

Can sometimes define probability(E) as |E| / | Ω|

Only works if all events are equally likely

Suppose P is a probability function

The sum of P(x) = 1

x is a subset of Ω

P(x1 or x2) = P(x1) + P(x2)

Example:

Uniformly at random pick an element from 1 to n

Ω = {1 to n}

P(i) = 1/n since all elements are uniform probability

A probability function is uniform if the probability of a sample point is equal for every sample point

P(E) = |E| / | Ω|

Example:

Suppose we toss a fair coin n times

Ω = {0, 1}n

P(x) = 1/2n

P(E) = |E| / | Ω|

P(exactly 5 heads) = nC5 / 2n = n5 / 2n

Example:

Suppose we toss a biased coin

P(H) = 2/3, P(T) = 1/3

Ω = {0, 1}n

If x has L heads, P(x) = (2/3)L \* (1/3)n-L

Suppose we want exactly L heads

P(E) = nCL \* (2/3)L \* (1/3)n-L

Example:

Suppose we have a set S = 1 to n

Uniformly at random, pick a number from S = X, and a number = Y (with replacement)

Ω = (1 – n) \* (1 – n)

P(x) = 1/n2

Example:

Suppose we want the probability that we do the experiment above, and all but one of the outcomes is 5

P(E) = (2n-1) / n2

For large samples, this is roughly equal to 2/n

If events are disjointed, P(e1 or e2) = P(e1) + P(e2)

Example:

Suppose we toss a fair coin n times

What is P(we see at least 2logn consecutive heads)?

**Write down later**

**Independence:**

If A and B are two events, they are independent if P(A and B) = P(A) \* P(B)